

The Static Quantum Multiverse

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Abstract

We consider the multiverse in the intrinsically quantum mechanical framework recently proposed in Refs. [1, 2]. By requiring that the principles of quantum mechanics are universally valid and that physical predictions do not depend on the reference frame one chooses to describe the multiverse, we find that the multiverse state must be static—in particular, the multiverse does not have a beginning or end. We argue that, despite its naive appearance, this does not contradict observation, including the fact that we observe that time flows in a definite direction. Selecting the multiverse state is ultimately boiled down to finding normalizable solutions to certain zero-eigenvalue equations, analogous to the case of the hydrogen atom. Unambiguous physical predictions would then follow, according to the rules of quantum mechanics.

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1 Introduction

The goal of fundamental physics is to find a prescription in which (potentially) testable predictions can be made and, through it, to learn how nature works at the most fundamental level. In the present way physics is formulated, this can be done in three steps:

- (i) “*Theory*” — We must specify the fundamental structure of the theory, which consists of the following two parts:
 - (i-1) *Kinematics* — We must understand what is a “state” which somehow represents the status of a physical system. We must also understand how it is related to the observed reality. For example, in conventional quantum mechanics a state is a ray in Hilbert space, which is related to reality through the Born rule, while in classical mechanics a state is a point in classical phase space (so is directly observable).
 - (i-2) *Dynamics* — We must know a (set of) fundamental law(s) the states obey. In quantum mechanics it is the Schrödinger equation, $i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$, while in classical mechanics it is the Newton equation, $m\ddot{\mathbf{x}}(t) = \mathbf{F}$.
- (ii) “*System*” — We then need to specify a system we consider, which again consists of two parts:
 - (ii-1) *Kinematics* — We need to know the kinematical structure of the system. In quantum mechanics this corresponds to specifying the Hilbert space, which is characterized by its dimension and operators acting on its elements. In classical mechanics, it is given by the dimension of the phase space.
 - (ii-2) *Dynamics* — We also need to specify dynamics of the system. In the examples in (i-2), we need to give the forms of H and \mathbf{F} , respectively.
- (iii) “*Selection Conditions*” — Even if (i) and (ii) are known, we still need to provide “selection conditions” on a state. Usually, they are given in the form of boundary conditions, for example as the knowledge one already has, e.g. $|\Psi(0)\rangle$ and $\{\mathbf{x}(0), \dot{\mathbf{x}}(0)\}$, before making predictions on something unknown, e.g. $|\Psi(t)\rangle$ and $\{\mathbf{x}(t), \dot{\mathbf{x}}(t)\}$ for $t > 0$.

To understand the ultimate structure of nature, we would want to do the above in the context of cosmology, and see whether the resulting predictions are consistent with what we observe. In this respect, physics of eternal inflation—which occurs under rather general circumstances [3]—has caused tremendous confusions in recent years. A major problem has been the so-called measure problem: even if we know the initial state and its subsequent evolution, we cannot define (even probabilistic) predictions unambiguously.¹ This occurs because in eternal inflation anything that can happen will happen infinitely many times, so it apparently leads to arbitrariness in predictions,

¹There are several varying, though related, definitions of the measure problem in literature. In this paper we adopt the definition as stated here.

associated with how these infinities are regularized [4]. Such an arbitrariness would prevent us from making well-defined predictions, so it seemed that to define the theory we needed to specify the exact way of regulating spacetime, where the infinities occur. This would be quite uncomfortable, since then the theory requires a specification of a (ad hoc) regularization prescription *beyond the basic principles of quantum mechanics and relativity*.

Recently, a framework that addresses this problem has been proposed in Refs. [1, 2], which allows for an intrinsically quantum mechanical treatment of the eternally inflating multiverse (see Ref. [5] for a review directed to a wide audience). In this framework, physics is described in a *fixed reference (local Lorentz) frame* associated with a fixed reference point p , with spacetime existing only within its (stretched) apparent horizon. An essential point is that the principles of quantum mechanics constrain the space of states \mathcal{H}_{QG} [2] in such a way that the problem of infinity does not arise. Namely, the correct identification of (ii-1) avoids the problem, without changing (i) from that of usual unitary quantum mechanics. A state representing the multiverse $|\Psi(t)\rangle$ “evolves” deterministically and unitarily in \mathcal{H}_{QG} , following the laws of quantum mechanics: $i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$. Here, t is an auxiliary parameter introduced to describe the “evolution” of the state, and need not be directly related to physical time we observe. Once the state $|\Psi(t)\rangle$ is known, physical predictions can be obtained through the (extended) Born rule [1, 2] without suffering from an infinity or ambiguity. This framework makes it possible that once a boundary condition on the state, e.g. $|\Psi(t)\rangle$ at some $t = t_0$, is given (element (iii)) and the explicit form of H acting on \mathcal{H}_{QG} is understood, e.g. by studying string theory (element (ii-2)), then unambiguous predictions are obtained for any physical questions one asks. While the framework does not achieve all of (i)–(iii), it does eliminate the ambiguity associated with the measure problem and provides a setting in which the remaining issues can be discussed.

In this paper we consider the issue of (iii) in the quantum mechanical framework of the multiverse described above. We take the following hypothesis:

Hypothesis I: The laws of quantum mechanics are not violated.

This—in particular the fact that the evolution of a quantum state is deterministic and unitary—implies that the multiverse state exists all the way from $t = -\infty$ to $+\infty$. Namely, the multiverse does not have a beginning or end. (For recent discussions on the beginning of the eternally inflating multiverse, see, e.g., Refs. [6].) There are three potential issues in this picture:

- *Uniqueness* — What is the selection condition imposed on the multiverse state, on which physical predictions will depend? In particular, what is the principle determining it?
- *Well-definedness* — The (extended) Born rule formula in general involves t integrals, which would now run from $-\infty$ to $+\infty$. Will this give well-defined probabilities?

- *Consistency* — Are the resulting predictions consistent with observation? In particular, are they consistent with the observed arrow of time, even if there is no beginning or end?

In this paper we argue that consistency with observation excludes the possibility that the selection condition is determined purely in \mathcal{H}_{QG} , without referring to an operator algebra. In particular, this excludes the possibility that the multiverse is in the maximally mixed state in \mathcal{H}_{QG} . We then propose that the multiverse state must satisfy the following simple criterion:

Hypothesis II: Physical predictions do not depend on the reference frame one chooses.

We show that this requirement leads to the condition

$$\frac{d}{dt} |\Psi(t)\rangle = 0 \quad \Leftrightarrow \quad H |\Psi(t)\rangle = 0, \quad (1)$$

where we have taken t to be the proper time at p ; namely, we find that *the multiverse state must be static!* We will argue that despite its naive appearance, this does not contradict observation, including the fact that we observe that time flows in a definite direction. It simply gives constraints on the structure of H , on which we will allow for making arbitrary assumptions, given that its explicit form is not available under current theoretical technology. We will also argue that the hypothesis leads to unique and well-defined predictions for any physical questions, once one knows the explicit form of H (element (ii-2) listed at the beginning). Specifically, any physical question can be phrased in the form: given what we know A about a state, what is the probability for it to be consistent also with B ? And the relevant probability is given by

$$P(B|A) = \frac{\langle \Psi | \mathcal{O}_{A \cap B} | \Psi \rangle}{\langle \Psi | \mathcal{O}_A | \Psi \rangle}, \quad (2)$$

where $|\Psi\rangle \equiv |\Psi(0)\rangle$, and \mathcal{O}_X is the operator projecting onto states consistent with condition X .

There are two comments. First, given Hypothesis I, Hypothesis II arises as a consequence of general covariance (and its suitable extension to the quantum regime) if we assume that the multiverse is in a zero-eigenvalue eigenstate of global energy and boost operators. This condition, therefore, provides another, more technical way of stating Hypothesis II. Second, without knowledge of the ultimate structure of H in quantum gravity, the scenario presented here is not the only option available within the framework of Refs. [1, 2], although it seems to be the most natural possibility. For example, one might imagine that the multiverse has a “beginning,” and evolves only thereafter. (This violates both Hypotheses I and II.) The framework of Refs. [1, 2] itself may still be applied in such a case.

The organization of this paper is as follows. In the next section, we review the framework of the quantum multiverse given in Refs. [1, 2], and discuss the issue of selection conditions in that context. In Section 3, we reconsider what the arrow of time is. We emphasize that the

observed flow of time does not necessarily mean that the state is actually evolving. In Section 4, we explore the possibility that the selection condition is expressed in \mathcal{H}_{QG} without referring to any quantum operator. We find that this forces the multiverse to be in the maximally mixed state in \mathcal{H}_{QG} , which is observationally excluded. In Section 5, we present our main scenario in which the multiverse state is determined by the two hypotheses described above. We find this implies that the multiverse state must be static, and discuss how it can be realized in the cosmological context. We also see that the scenario arises as a consequence of quantum mechanics and general covariance if we assume that the multiverse is in a zero-eigenvalue eigenstate of global energy and boost operators. In Section 6, we discuss the consistency of the scenario with observation, specifically the observed arrow of time. In Section 7, we provide our final discussions. We draw a close analogy of the present scenario with the case of the hydrogen atom, underscoring the intrinsically quantum nature of the scenario.

2 Framework—the Quantum Multiverse

In this section we review the framework of Refs. [1, 2], describing the quantum multiverse. We also discuss the issue of selection conditions in making predictions within this framework.

2.1 The Hilbert space

The framework is based on the principles of quantum mechanics. In particular, we formulate it using Hamiltonian (canonical) quantum mechanics, although the equivalent Lagrangian (path integral) formulation should also be possible. We take the Schrödinger picture throughout.

Recall that to do Hamiltonian quantum mechanics, we need to fix all gauge redundancies. Since these redundancies include coordinate transformations in a theory with gravity, states must be defined *as viewed from a fixed (local Lorentz) reference frame* associated with a fixed reference point p . Moreover, to avoid violation of the principles of quantum mechanics, they must represent only spacetime regions within the (stretched) apparent horizons of p , as suggested first in the study of black hole physics [7]. Together with the states associated with spacetime singularities, these states form the Hilbert space for quantum gravity \mathcal{H}_{QG} .

The construction of \mathcal{H}_{QG} can proceed analogously to the usual Fock space construction in quantum field theory. For a set of fixed semi-classical geometries $\mathcal{M} = \{\mathcal{M}_i\}$ having the same apparent horizon $\partial\mathcal{M}$, the Hilbert space is given by

$$\mathcal{H}_{\mathcal{M}} = \mathcal{H}_{\mathcal{M},\text{bulk}} \otimes \mathcal{H}_{\mathcal{M},\text{horizon}}, \quad (3)$$

where $\mathcal{H}_{\mathcal{M},\text{bulk}}$ and $\mathcal{H}_{\mathcal{M},\text{horizon}}$ represent Hilbert space factors associated with the degrees of freedom inside and on the horizon $\partial\mathcal{M}$. The dimensions of these factors are both $\exp(\mathcal{A}_{\partial\mathcal{M}}/4)$, where $\mathcal{A}_{\partial\mathcal{M}}$

is the area of the horizon in Planck units:

$$\dim \mathcal{H}_{\mathcal{M}} = \dim \mathcal{H}_{\mathcal{M},\text{bulk}} \times \dim \mathcal{H}_{\mathcal{M},\text{horizon}} = \exp\left(\frac{\mathcal{A}_{\partial\mathcal{M}}}{2}\right), \quad (4)$$

consistently with the holographic principle [8]. The full Hilbert space for dynamical spacetime is then given by the direct sum of the Hilbert spaces for different \mathcal{M} 's

$$\mathcal{H} = \bigoplus_{\mathcal{M}} \mathcal{H}_{\mathcal{M}}. \quad (5)$$

In addition, the complete Hilbert space for quantum gravity must contain “intrinsically quantum mechanical” states, associated with spacetime singularities [2]:

$$\mathcal{H}_{\text{QG}} = \mathcal{H} \oplus \mathcal{H}_{\text{sing}}, \quad (6)$$

where $\mathcal{H}_{\text{sing}}$ represents the Hilbert space for the singularity states. The evolution of the multiverse state $|\Psi(t)\rangle$, which represents the entire multiverse, is deterministic and unitary in \mathcal{H}_{QG} , but not in $\mathcal{H}_{\mathcal{M}}$ or \mathcal{H} .

The dimension of the complete Hilbert space \mathcal{H}_{QG} is infinite, as the dimensions of Hilbert subspaces associated with stable Minkowski space and spacetime singularities are infinite:

$$\dim \mathcal{H}_{\text{Minkowski}} = \infty, \quad \dim \mathcal{H}_{\text{sing}} = \infty. \quad (7)$$

This implies, by the second law of thermodynamics, that a *generic* multiverse state in \mathcal{H}_{QG} will evolve at large t into a superposition of terms corresponding to supersymmetric Minkowski space or spacetime singularity:

$$|\Psi(t)\rangle \xrightarrow{t \rightarrow \infty} \sum_i a_i(t) |\text{supersymmetric Minkowski space } i\rangle + \sum_j b_j(t) |\text{singularity state } j\rangle, \quad (8)$$

where we have assumed that the only absolutely stable Minkowski vacua are supersymmetric ones, as suggested by the string landscape picture [9].

Note that an infinite number of states exist only in a Hilbert subspace associated with a spacetime singularity or a Minkowski space *in which the area of the apparent horizon diverges* $\mathcal{A}_{\partial\mathcal{M}} = \infty$. In particular, the number of states associated with a fixed Friedmann-Robertson-Walker (FRW) time in a Minkowski bubble is finite for any finite energy density ρ , since the area of the apparent horizon is given by $\mathcal{A}_{\partial\mathcal{M}} = 3/2\rho$ (with ρ in Planck units) [10], so that $\mathcal{A}_{\partial\mathcal{M}} < \infty$ for $\rho > 0$.

2.2 The (extended) Born rule

For a given multiverse state $|\Psi(t)\rangle$, physical predictions can be obtained following the rules of quantum mechanics. An important point is that the “time” parameter t here is simply an auxiliary parameter introduced to describe the “evolution” of the state. The physical information is only in *correlations* between events; specifically, time evolution of a physical quantity X is nothing more than a correlation between X and a quantity that can play the role of time, such as the location of the hands of a clock or the average temperature of the cosmic microwave background in our universe. A particularly useful choice for t is the proper time at p , which we will assume for the rest of the paper.

Any physical question can then be phrased as: given what we know A about a state, what is the probability for that state to be consistent also with condition B ? In the context of the multiverse, this probability is given by [1]

$$P(B|A) = \frac{\int dt \langle \Psi(0) | U(0, t) \mathcal{O}_{A \cap B} U(t, 0) | \Psi(0) \rangle}{\int dt \langle \Psi(0) | U(0, t) \mathcal{O}_A U(t, 0) | \Psi(0) \rangle}, \quad (9)$$

where $U(t_1, t_2) = e^{-iH(t_1-t_2)}$ is the “time evolution” operator with H being the Hamiltonian of the entire system for a fixed “time” parameterization t (here the proper time at p), and \mathcal{O}_X is the operator projecting onto states consistent with condition X . Note that since we have already fixed a reference frame, conditions A and B in general must involve specifications of ranges of location and velocity in which a physical object must be with respect to the reference point p .

As we will discuss in more detail in Section 3, the formula in Eq. (9) can be used to answer any physical questions including those about dynamical evolution of a system, despite the fact that conditions A and B both act at the same moment t . We therefore base all our discussions on Eq. (9) in this paper. (For a different formula that can be used more easily in many practical contexts, see Ref. [2].) The t integrals in the equation run over the entire region under consideration. Suppose, for example, that we know the universe/multiverse is in a particular, e.g. eternally inflating, state $|\Psi(0)\rangle$ at $t = 0$, and want to predict what happens in $t > 0$. In this case, the integrals must be taken from $t = 0$ to ∞ , since condition A may be satisfied at any value of $t > 0$ in some component of $|\Psi(t)\rangle$. Note that despite the integrals running to ∞ the resulting probability is well-defined, because Eq. (8) prohibits an event from occurring infinitely many times with a finite probability, which would cause divergences.

2.3 The issue of selection conditions

What kind of predictions does the framework described above allow us to make? While the framework addresses the issues of infinity and the ambiguity associated with it (i.e. the measure problem as defined here), it is certainly not complete. In particular, ...

- (a) “*Unspecified System*” — We did not identify the system *explicitly*. Specifically, the complete theory of quantum gravity is not known, so that we do not know the form of H , especially the part acting on the horizon degrees of freedom. This particular issue can be bypassed if we focus only on questions addressed at the semi-classical level. Even then, however, current technology does not give us the explicit form of H , e.g. the structure of the string landscape.
- (b) “*Selection Conditions*” — Predictions in general depend on the selection condition we impose on $|\Psi(t)\rangle$ (even if we know H explicitly). For example, in the situation considered at the end of the previous subsection, they depend on the initial condition $|\Psi(0)\rangle$.

These limitations may still allow us to make certain predictions, possibly with some assumption on the dynamics of the system. First of all, if we are interested in a system localized in a small region compared with the horizon scale, then we can make predictions on the evolution of the system (i.e. correlation with a physical quantity that plays the role of time) using prior information about the system—indeed, one can show that Eq. (9) is reduced to the standard Born rule in such a case. Second, if we are interested in quantities whose distributions in H are reasonably inferred in an anthropically allowed range, then we can predict the probability distribution of these quantities seen by a typical observer, under the assumption that the selection condition provides a statistically uniform prior [11]. This is, for example, the case if we are interested in the probability distribution of the cosmological constant one observes [12].

However, if we want to answer general “multiversal” questions, e.g. if we want to predict the probability distribution of the structure of the low-energy Lagrangian found by an intellectual observer in the multiverse, then we would need to address both (a) and (b) above. (What the intellectual observer means can be specified explicitly by condition A .) For (a), one could hope that future progress, e.g. in string theory, might provide us (at least the relevant information on) the form of H in \mathcal{H}_{QG} . But what about (b)?

There are at least three aspects which make this problem substantial:

- One might speculate that a physical theory only allows for relating a given initial state to another final state, which is indeed the case in conventional Newtonian and quantum mechanics. In the present context, this implies that to make general predictions, we need to know the state $|\Psi(t)\rangle$ explicitly for some t . This is, however, impossible to do observationally! Quantum mechanics does not allow us to know the exact state *including us, the observer*. Moreover, $|\Psi(t)\rangle$ is the quantum state for the whole multiverse, so it in general contains terms representing different semi-classical universes than what we live in.
- General predictions in the multiverse, therefore, will be possible only if we have a *theoretical* input on the selection condition of $|\Psi(t)\rangle$. Suppose it takes the form of a specific “initial condition,” $|\Psi(0)\rangle$. Then, the predictions depend on $|\Psi(0)\rangle$, so that, unless we have a separate

theory of the initial condition, the uniqueness of (even statistical) predictions will be lost.

- Imagine that there is, indeed, a theory of the initial condition giving a particular state $|\Psi(0)\rangle$, and that the framework described in Sections 2.1 and 2.2 applies only to $t > 0$. In this case, the laws of quantum mechanics, especially deterministic and unitary evolution of the state, is violated at $t = 0$. While this is possible, it would be more comfortable if fundamental principles, such as those of quantum mechanics, do not have an “exception” like this.

In the rest of the paper, we will address the problem of selection conditions, i.e. issue (b), from the viewpoint of extrapolating the principles of quantum mechanics to the maximum extent possible. By postulating a certain simple criterion, and requiring consistency with observation, we will arrive at the picture that the multiverse state must, in fact, be static. This provides a strong selection of the possible states. The observed flow of time arises from the structures of H in \mathcal{H}_{QG} , and not because of a t dependence of $|\Psi(t)\rangle$.

3 The Observational “Data”

Any selection condition imposed on the multiverse state must not lead to results inconsistent with observation, if it is to do with nature. The basic observational fact in our universe is that we see time flow in a definite direction, and predictions of a theory must not contradict it. As we will see, this seemingly weak requirement, in fact, provides a powerful tool to determine the selection condition. Here, we carefully consider what the observed flow of time actually means in the context of the quantum multiverse.

3.1 What is the arrow of time?

What does the fact that we see time flow really mean? At the most elementary level, it just means that the memory state of my (or your) brain is consistent with the hypothesis that it is generated by an environment whose coarse-grained entropy evolves from lower to higher values. The point is that the states consistent with such a hypothesis are very special ones among all the possible states the brain can take. What the fundamental theory must explain is why my brain is in one of these highly exceptional states.

To illustrate the basic idea further, let us consider a more corporeal example of a chair in a room. Suppose you are looking at only a half of the scene and find a half of a chair and of a room there; see the upper picture in Fig. 1. What would you expect to be in the other half? In the ordered world we live in, we expect to see the other half of the chair and the room, possibly with some other things such as a painting on the wall, as depicted in the lower left picture in Fig. 1. However, any such configurations are extremely rare among all the possible configurations

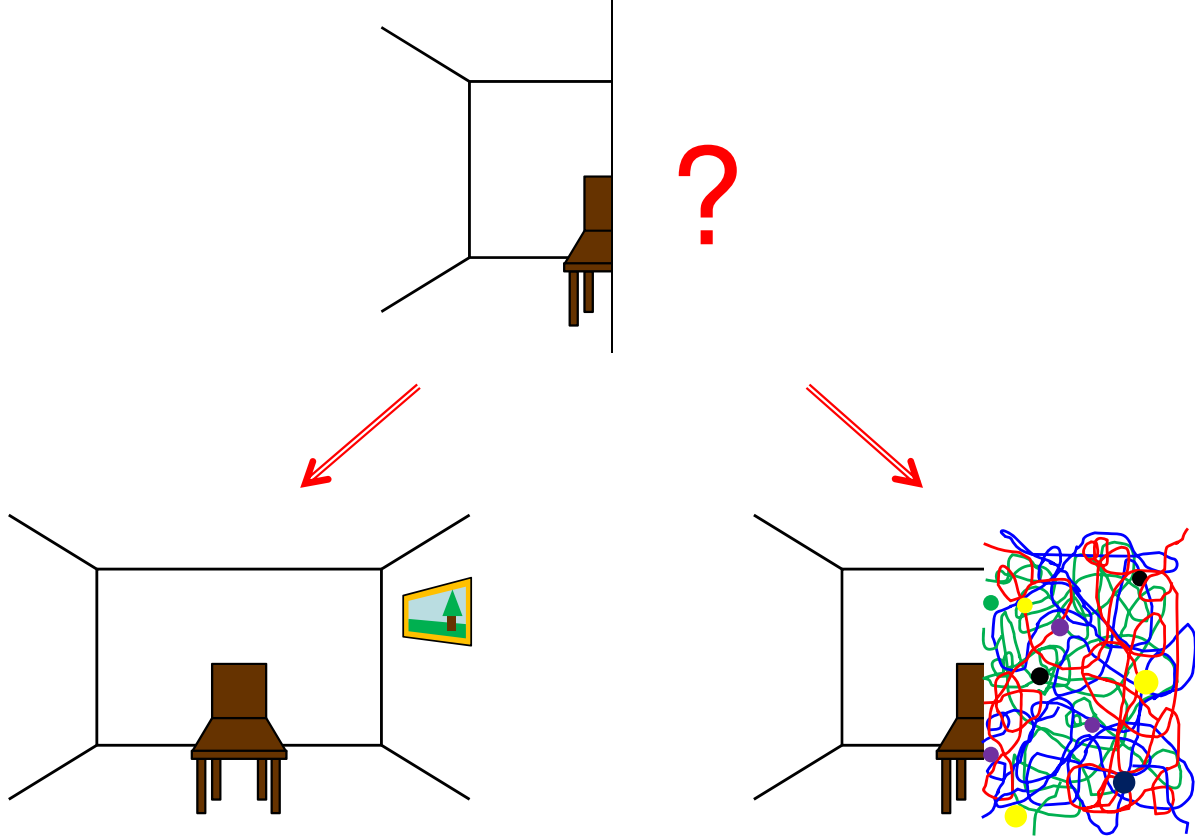


Figure 1: Suppose you know that there are a half of a chair and of a room in the first half of the scene (the upper picture). In a regular ordered world, you expect the second half of the scene contains the other half of the chair and the room, possibly with some other things (the lower left picture). On the other hand, the number of such states is much smaller than that of states in which the second half contains random, disordered configurations (the lower right picture).

physically allowed and consistent with the first half of the scene. The vast majority of these general configurations correspond to the ones in which the other half of the scene is completely disordered, as depicted in the lower right picture in Fig. 1. The arrow of time refers to the fact that we always find ordered configurations (as in the lower left picture) rather than disordered ones (as in the lower right picture) in any similar situation, i.e. not only for a chair in a room but also for other objects. Such ordered configurations can be naturally expected if the entire system is evolved from a state having a much lower coarse-grained entropy; otherwise, we would expect disordered ones since the number of states corresponding to disordered configurations is much larger than that corresponding to ordered ones.

In the context of the multiverse, the fact that we live in our universe and see the arrow of time tells us two things:

- (A) A typical observer among all the “conscious” observers in the multiverse (including fluke, Boltzmann brain observers [13]) must live in a universe consistent with our current knowledge, i.e. a universe whose low energy physics is described by the standard model of particle physics and cosmology.
- (B) When we ask any conditional probability $P(B|A)$ within our universe, i.e. when precondition A is chosen such that it selects a situation in our universe (e.g. my brain state), the answer should be dominated by one that arises from a low coarse-grained entropy state through evolution.

These two are the only things we definitely know from observation about the structure of the multiverse; for example, the arrow of time may not exist in other universes, i.e. the probabilities may be dominated by disordered configurations in those universes. What we must require is that the theory must (at least) be compatible with these two conditions.

The above discussion shows that the following two statements are literally *equivalent* as concepts: “An observer sees the arrow of time” and “There is no Boltzmann brain problem.” This is consistent with the picture presented recently by Bousso [14], who analyzed the arrow of time in the context of the evolving multiverse in the landscape. Historically, the argument like the one here was first used to exclude the possibility that our universe, which has a positive cosmological constant, is absolutely stable [13]. It was also argued in Ref. [2] that it excludes the possibility that the multiverse is a closed, finite system if it has a *generic* initial condition in \mathcal{H}_{QG} . This possibility, however, is allowed if the selection condition imposed on the entire multiverse state is special, as is the case in the scenario considered in this paper.

In summary, a selection condition imposed on the multiverse state must be such that the resulting probabilities are consistent with conditions (A) and (B) listed above. In particular, this leads to the following corollary:

- (*) Any selection condition on $|\Psi(t)\rangle$ that leads to an (almost) equal probability for all the possible states in \mathcal{H}_{QG} corresponding to our universe is observationally excluded.

This is because such a scenario would lead to the probabilities being dominated by disordered configurations in our universe, contradicting observation. This condition will play an important role in rejecting a possible selection condition in Section 4.

3.2 Is the multiverse really evolving?

The consideration given above also illuminates the following question: is the multiverse really evolving? The answer is: it need not. In order to be consistent with the observed arrow of time, it is only necessary that the probabilities *in our universe* are dominated by configurations that are consistent with the hypothesis that the system has evolved from a lower coarse-grained

entropy state. This, however, does not necessarily mean that the multiverse state $|\Psi(t)\rangle$ is actually evolving in t . It simply says that the probabilities obtained from $|\Psi(t)\rangle$ should be consistent with the hypothesis that our universe has evolved from a lower entropy state.

One might think that we actually “witnessed” that the state evolved as we came into being and grew. The interpretation of this fact, however, needs care—all we know is that our memory states are such that they are *consistent with* those obtained by interacting with environments that evolve from lower to higher entropy states. Similarly, we usually consider that our universe has evolved from the early big-bang, but all we really know is that the current state of the universe is consistent with the hypothesis that it has evolved from a lower entropy, big-bang state. As we have seen in the previous subsection, what these observations are really telling us is that in our universe different parts of physical configurations are correlated in certain (very) special ways. They do not mean that the multiverse state $|\Psi(t)\rangle$ *must* be evolving.

The question of whether a physical system is viewed as evolving or not, therefore, can be determined by asking questions about a “current” configuration, i.e. configuration at a fixed value of t . If the configuration is consistent with the hypothesis that the system has evolved from a lower entropy state, then we *interpret* it as the system evolving—it is not necessary that the state itself is actually changing with t . To do such a determination, it is enough to use the formula of Eq. (9), in which conditions A and B act at the same moment. In fact, in quantum mechanics, when we obtain information about a system we do that indirectly by observing imprints in the environment left by the system [15], so this is almost exactly what we do in reality when we study the “history” of a system.

Summarizing, the observed flow of time does not require that the multiverse state is actually changing with t . It simply requires that the resulting probabilities satisfy the two conditions described in the previous subsection: (A) and (B). The probability formula in which conditions A and B both act at the same moment can be used to answer any physical questions, including those about a system that we interpret as dynamically evolving.

4 Selection Conditions and Operators

We now start exploring possible selection conditions that can be imposed on the multiverse state. As stated in the introduction, we consider that the laws of quantum mechanics are not violated (Hypothesis I), which forces the multiverse state to exist for all values of t : from $-\infty$ to $+\infty$. This implies that, once a selection condition is given at a particular moment, which we take as $t = 0$, then the state is uniquely determined by solving the Schrödinger equation both forward and backward in t .

In this section, we ask the following question: can the selection condition be given in Hilbert

space \mathcal{H}_{QG} without referring to any quantum operator? If this is possible, then it would imply that the form of the selection condition, written purely in terms of quantum states, must be basis independent, since we cannot specify a basis without knowledge of operators and how they act on elements in the Hilbert space. (Note that Hilbert space itself does not contain any physical information except for its dimension, i.e. any complex Hilbert spaces having the same dimension are identical with each other.) We will see that there is only one possible selection condition satisfying this criterion, and that it is observationally excluded. We will therefore learn that the expression for the selection condition in \mathcal{H}_{QG} must involve some information about the quantum operators.

4.1 The selection condition without an operator

Suppose that the multiverse is in a pure state, and that the selection condition at $t = 0$ is given by

$$|\Psi(0)\rangle = \sum_i c_i |\alpha_i\rangle, \quad (10)$$

where $|\alpha_i\rangle$ represents a complete, orthonormal basis for the elements in \mathcal{H}_{QG} , and c_i are fixed coefficients characterizing the selection condition. Can the expression in Eq. (10)—*including the values of c_i* —be basis independent?

Consider that we perform an arbitrary basis change

$$|\alpha_i\rangle = \sum_j U_{ij} |\alpha'_j\rangle, \quad (11)$$

where U_{ij} is an arbitrary unitary matrix. In the new basis, the expression in Eq. (10) is written as $|\Psi(0)\rangle = \sum_i c'_i |\alpha'_i\rangle$, where the new coefficients c'_i are given by $c'_i = \sum_j c_j U_{ji}$. In order for the form of the selection condition to be basis independent, we need to have

$$c_i = c'_i = \sum_j c_j U_{ji} \quad (12)$$

for an arbitrary U_{ij} . This condition cannot be satisfied unless $c_i = 0$ for all i . Therefore, it is not possible to write a selection condition without referring to any quantum operator if the multiverse state is pure.

Suppose now that the multiverse is in an intrinsically mixed state, which takes the form

$$\rho(0) = \sum_{i,j} d_{ij} |\alpha_i\rangle \langle \alpha_j| \quad (13)$$

at $t = 0$, where d_{ij} is a positive semi-definite Hermitian matrix. The basis change in Eq. (11) then leads to $\rho(0) = \sum_i d'_{ij} |\alpha'_i\rangle \langle \alpha'_j|$, where the new coefficients are given by $d'_{ij} = \sum_{k,l} U_{ik} d_{kl} U_{jl}^*$. In

order for the selection condition to be basis independent, we must have

$$d_{ij} = d'_{ij} = \sum_{k,l} U_{ik} d_{kl} U_{jl}^* \quad (14)$$

for an arbitrary U_{ij} . This has the unique solution (up to the overall coefficient):

$$d_{ij} \propto \delta_{ij}. \quad (15)$$

We thus find that the requirement is satisfied if the multiverse state is specified by

$$\rho(0) \propto \sum_i |\alpha_i\rangle \langle \alpha_i|, \quad (16)$$

namely if the multiverse is in the maximally mixed state in \mathcal{H}_{QG} at $t = 0$.

4.2 Can the multiverse be in the maximally mixed state?

Once the selection condition is given by Eq. (16), the multiverse state $\rho(t)$ for arbitrary t can be obtained using the evolution equation

$$\rho(t) = U(t, 0) \rho(0) U(0, t). \quad (17)$$

Since $\rho(0)$ is proportional to the unit matrix in \mathcal{H}_{QG} , however, this gives

$$\rho(t) = \rho(0), \quad (18)$$

i.e. the multiverse is in the maximally mixed state at all times.

Equations (16) and (18) imply that all the possible states in \mathcal{H}_{QG} corresponding to our universe are equally probable. This is exactly the possibility that is observationally excluded by corollary (*) in Section 3.1. Since we have arrived at this conclusion only by assuming that the selection condition is written without referring to a quantum operator in \mathcal{H}_{QG} , we learn that the condition must in fact involve a quantum operator. The significance of this result lies in the fact that in quantum mechanics, operators are the objects that contain information about the system—the condition imposed on the multiverse state must reflect the structure of the system.

5 The Static Quantum Multiverse

What operators can be used in the condition imposed on the multiverse state? Since the multiverse contains many universes in which low energy physical laws differ, they cannot be “vacuum specific” operators. In this section, we identify candidate operators—those generating reference frame changes and that generating evolution.

We then impose the requirement that physical predictions are independent of a reference frame one chooses to describe the multiverse (Hypothesis II in the introduction). We will see that this implies that the multiverse state is independent of t , i.e. it must be static. As discussed in Section 3.2, this does not necessarily contradict observation. (The consistency with the observed flow of time will be discussed further in Section 6.) We will also see that with Hypothesis I, Hypothesis II can be viewed as a consequence of requiring that the multiverse is in an eigenstate of global energy and boost operators with zero eigenvalues.

5.1 Reference frame changes

Recall that in the framework of Refs. [1, 2], quantum states allowing for spacetime interpretation, i.e. elements of $\mathcal{H} \subset \mathcal{H}_{\text{QG}}$, represent only the spacetime regions inside and on the (stretched) apparent horizons as viewed from a fixed reference frame associated with a fixed reference point p . What happens if we change the reference frame?

Consider a state representing a configuration in de Sitter space. If we perform a spatial translation, which is equivalent to shifting the location of p , then it will necessarily mix the degrees of freedom inside and on the horizon because the state is defined only in the restricted spacetime region. This is precisely the phenomenon we call the observer dependence of the horizon: (some of) the degrees of freedom associated with internal space for one observer are described as those associated with the horizon by another. Next, consider a state which will later form a black hole, with p staying outside of the black hole horizon. Such a state will not contain the spacetime region inside the black hole horizon because it will be outside p 's horizon. Now, imagine that we change the reference frame by performing a boost at an early time so that p will be inside the black hole horizon at late times. In this new frame, the state at late times *does* contain the spacetime region inside the black hole horizon, although now it does *not* contain Hawking radiation quanta escaping to the future null infinity, which were included in the state before performing the reference frame change. This is exactly the phenomenon of black hole complementarity [7]. The present framework, therefore, allows us to understand the two phenomena described above in a unified manner as special cases of general reference frame changes [2]; in particular, the concept of spacetime depends on the reference frame.

As any symmetry transformation, reference frame changes must be represented by unitary transformations acting on Hilbert space \mathcal{H}_{QG} . What is the set of generators representing these transformations, and what is the algebra they satisfy?

In the limit $G_N \rightarrow 0$, the set of transformations associated with the reference frame changes and a shift of the origin of t (time translation) is reduced to the standard Poincaré transformations, which is analogous to the fact that the standard Poincaré group is reduced to the Galilean group in the limit $c \rightarrow \infty$ [2]. Here, G_N and c are Newton's constant and the speed of light, respectively.

In the case of the reduction associated with $c \rightarrow \infty$, the structure of infinitesimal transformations changes. This is seen clearly in the Poincaré algebra:

$$\begin{aligned}
[J_{[ij]}, J_{[kl]}] &= i (\delta_{ik} J_{[jl]} - \delta_{il} J_{[jk]} - \delta_{jk} J_{[il]} + \delta_{jl} J_{[ik]}), \\
[J_{[ij]}, K_k] &= i (\delta_{ik} K_j - \delta_{jk} K_i), \quad [K_i, K_j] = -\frac{i}{c^2} J_{[ij]}, \\
[J_{[ij]}, P_k] &= i (\delta_{ik} P_j - \delta_{jk} P_i), \quad [K_i, P_j] = \frac{i}{c^2} \delta_{ij} H, \quad [P_i, P_j] = 0, \\
[J_{[ij]}, H] &= [P_i, H] = [H, H] = 0, \quad [K_i, H] = iP_i,
\end{aligned} \tag{19}$$

where $J_{[ij]}$, K_i , and P_i are the generators of spatial rotations, boosts, and spatial translations, respectively, and we have exhibited c explicitly. This algebra is reduced to a different algebra, i.e. that of the Galilean group, as $c \rightarrow \infty$:

$$\begin{aligned}
[J_{[ij]}, J_{[kl]}] &= i (\delta_{ik} J_{[jl]} - \delta_{il} J_{[jk]} - \delta_{jk} J_{[il]} + \delta_{jl} J_{[ik]}), \\
[J_{[ij]}, K_k] &= i (\delta_{ik} K_j - \delta_{jk} K_i), \quad [K_i, K_j] = 0, \\
[J_{[ij]}, P_k] &= i (\delta_{ik} P_j - \delta_{jk} P_i), \quad [K_i, P_j] = i\delta_{ij} M, \quad [P_i, P_j] = 0, \\
[J_{[ij]}, H] &= [P_i, H] = [H, H] = 0, \quad [K_i, H] = iP_i,
\end{aligned} \tag{20}$$

where we have rescaled $H \rightarrow c^2 M + H$ to allow for the possibility that the original H has a constant piece that goes as c^2 . Can the algebra corresponding to the reference frame changes and time translation have extra terms beyond Eq. (19) that disappears in the limit $G_N \rightarrow 0$?

One can immediately see that it cannot. The generators of the reference frame changes consist of $J_{[ij]}$, K_i , and P_i , while that of time translation is H . Taking natural units, the mass dimensions of these generators are $[J_{[ij]}] = [K_i] = 0$ and $[P_i] = [H] = 1$, while that of Newton's constant is $[G_N] = -d + 2$, where d is the number of spacetime dimensions. It is then easy to find that for $d \geq 4$, where gravity is dynamical, there is no term one can add to the commutators in Eq. (19) that is linear in generators and has a positive integer power of G_N .² The algebra for the reference frame changes and time translation, therefore, is the same as that of the Poincaré transformations in Eq. (19). The effect of nonzero G_N appears as the reduction of the Hilbert space, but not in the transformation generators of the Poincaré group.

5.2 Selecting the multiverse state

Let us now require that predictions do not depend on the reference frame one chooses to describe the multiverse (Hypothesis II). Physically, this implies that there is neither absolute center nor the frame of absolute rest in the multiverse.

²For $d = 3$, one can add terms $\Delta[J, K_i] = i\gamma G_N P_i$ and $\Delta[K_1, K_2] = -i\gamma G_N H$, where $J \equiv J_{[12]}$ and γ is a real constant, without violating Jacobi identities. The significance of this is not clear.

Formally, our requirement can be stated as follows. Suppose we want to make physical predictions using projection operators \mathcal{O}_X , e.g. $X = A$, $A \cap B$, and so on. The relevant matrix elements are then $\langle \Psi(t) | \mathcal{O}_X | \Psi(t) \rangle$. Now, consider a multiverse state as viewed from a different reference frame: $|\Psi'(t)\rangle = S |\Psi(t)\rangle$, where S is the unitary operator representing the corresponding reference frame change. Our requirement is then

$$\langle \Psi(t) | \mathcal{O}_X | \Psi(t) \rangle = \langle \Psi'(t) | \mathcal{O}_X | \Psi'(t) \rangle \quad (21)$$

for arbitrary S and \mathcal{O}_X . Note that the operator in the right-hand side is *not* $\mathcal{O}'_X = S \mathcal{O}_X S^\dagger$, but the same \mathcal{O}_X as in the left-hand side. This equation, therefore, has a nontrivial physical content, imposing constraints on the multiverse state. (If we had \mathcal{O}'_X in the right-hand side, then the equation would simply represent a basis change, and thus would be trivial.)

In order to satisfy Eq. (21), the multiverse state must satisfy $S |\Psi(t)\rangle \propto |\Psi(t)\rangle$, so that it must be a simultaneous eigenstate of operators $J_{[ij]}$, K_i and P_i .³ One can then easily see from Eq. (19) that this requires that the multiverse state is also an eigenstate of H , and that the eigenvalues under $J_{[ij]}$, K_i , P_i , and H are all zero. The fact that the multiverse state is an eigenstate of H with zero eigenvalue means that

$$\frac{d}{dt} |\Psi(t)\rangle = 0, \quad (22)$$

i.e. the multiverse state is static! We can therefore write it simply as $|\Psi\rangle \equiv |\Psi(t)\rangle = |\Psi(0)\rangle$. The conditions coming from Hypothesis II can then be summarized as

$$J_{[ij]} |\Psi\rangle = K_i |\Psi\rangle = P_i |\Psi\rangle = H |\Psi\rangle = 0. \quad (23)$$

This provides selection conditions for the multiverse state.

In fact, given Hypothesis I, the conditions in Eq. (23) follow from a standard procedure of quantizing a system with redundancies [16], if we assume that the multiverse state is invariant under the action of global energy and boost operators. In this procedure, any gauge redundancy, including general coordinate transformations, appears as a supplementary condition imposed on quantum states, which eliminates unphysical degrees of freedom from the states. Starting from a consistent, general covariant quantum theory of gravity (which is presumably string theory), the states are subject to a huge number of supplementary conditions, some of which will be used to reduce the number of degrees of freedom from that implied by local field theory to that suggested by the holographic principle, as in Eq. (4). (This implies that the number of constraints

³It is, in principle, possible that the predictions are reference frame independent because the multiverse is in an intrinsically mixed state that satisfies $S\rho(t)S^\dagger = \rho(t)$ at all t but each component $|\psi_i(t)\rangle$ in $\rho(t)$ is *not* a simultaneous eigenstate of all the S 's. This is, however, the case only if $\rho(t)$ is the maximally mixed state in \mathcal{H}_{QG} (because of Schur's lemma), which is observationally excluded as we saw in Section 4.2. We must therefore require that each pure-state component leads to reference-frame independent predictions even if the multiverse is in a mixed state.

is much larger than that of the standard constraints associated with classical general coordinate transformations [17].) In this bigger (more redundant) picture, the framework of Refs. [1, 2] corresponds to the scheme in which all the gauge redundancies are explicitly fixed, except for the ones associated with the reference frame changes. These residual redundancies, i.e. those of the reference frame changes, must then have their own supplementary conditions imposed on the states living in \mathcal{H}_{QG} .

To illustrate this in a simple example, let us consider a spacetime that admits rectilinear coordinates x_i in a constant t hypersurface. In terms of Hamiltonian and momentum densities, $\mathcal{H}(x)$ and $\mathcal{P}_i(x)$, the Hilbert space \mathcal{H}_{QG} then corresponds to the space of states in which the constraints of the form

$$\begin{aligned} \int x_i x_j \mathcal{H}(x) d^3x |\Psi\rangle &= \int x_i x_j x_k \mathcal{H}(x) d^3x |\Psi\rangle = \dots \\ &= \int x_i x_j \mathcal{P}_k(x) d^3x |\Psi\rangle = \int x_i x_j x_k \mathcal{P}_l(x) d^3x |\Psi\rangle = \dots = 0, \end{aligned} \quad (24)$$

as well as those associated with holography and complementarity, are *already* imposed; namely, the states in \mathcal{H}_{QG} satisfy these constraints by construction. On the other hand, the constraints of the form

$$\int \mathcal{H}(x) d^3x |\Psi\rangle = \int x_i \mathcal{H}(x) d^3x |\Psi\rangle = \int \mathcal{P}_i(x) d^3x |\Psi\rangle = \int x_i \mathcal{P}_j(x) d^3x |\Psi\rangle = 0 \quad (25)$$

are *not* imposed to obtain \mathcal{H}_{QG} , so they must still be imposed on the states in \mathcal{H}_{QG} . Now, the generators of time translation and the reference frame changes are given by

$$\begin{aligned} H &= \int \mathcal{H}(x) d^3x + \epsilon, & P_i &= \int \mathcal{P}_i(x) d^3x + p_i, \\ K_i &= \int x_i \mathcal{H}(x) d^3x + k_i, & J_{[ij]} &= \int (x_i \mathcal{P}_j(x) - x_j \mathcal{P}_i(x)) d^3x + j_{[ij]}, \end{aligned} \quad (26)$$

where we have included global energy ϵ and momentum p_i operators (and the corresponding quantities in K_i and $J_{[ij]}$) that represent possible contributions from surface terms. Such terms can indeed arise in asymptotically Minkowski space, and play the role of what we consider the total energy and momentum of the system [18].

Note that it is the effect of global energy ϵ that allows for any evolution of states in t in quantum gravity, because

$$|\Psi(t_1)\rangle = e^{-iH(t_1-t_2)} |\Psi(t_2)\rangle = e^{-i\epsilon(t_1-t_2)} |\Psi(t_2)\rangle, \quad (27)$$

so unless $|\Psi(t)\rangle$ is a superposition of terms that give different values of ϵ , the state is stationary. In this picture, our Hypothesis II corresponds to the assumption that the multiverse is an eigenstate of ϵ and k_i with vanishing eigenvalues:

$$\epsilon |\Psi\rangle = k_i |\Psi\rangle = 0, \quad (28)$$

in which case we immediately see that $|\Psi\rangle$ also has zero eigenvalues under p_i and $j_{[ij]}$, and that Eq. (23) follows from the constraints in Eq. (25) (and vice versa). An important point is that for a state in \mathcal{H}_{QG} , the surface terms reside on the (stretched) apparent horizon, so that Eq. (28) is the assumption about the structure of the theory on this surface. This is in the intrinsically quantum gravitational regime, over which we currently do not have good theoretical control.

The selection of possible multiverse states, therefore, is boiled down to solving the infinite-dimensional matrix equations in Eq. (23). Here, we assume that there is no other selection condition, i.e. Eq. (23) is enough to fully select the system. (We assume that other supplementary conditions, e.g. those associated with standard gauge symmetries, are already taken care of. Also, since all the redundancies associated with gravity other than those corresponding to the reference frame changes are supposed to be fixed in the present framework [1], there are no more conditions arising from considerations of gravity.) We look for solutions to Eq. (23) of the form

$$|\Psi\rangle = \sum_i c_i |\alpha_i\rangle, \quad \sum_i |c_i|^2 < \infty, \quad (29)$$

where $|\alpha_i\rangle$ represents a complete, orthonormal basis in \mathcal{H}_{QG} , so that the sums of i run to infinity; see Eq. (7). The normalizability condition here is imposed for the following (usual) reason. Suppose there are normalizable solutions $|\Psi_I\rangle$ ($I = 1, \dots, N$) satisfying Eq. (29), as well as non-normalizable solutions $|\Psi_I\rangle$ ($I = N + 1, \dots, K$). The non-normalizable solutions will have coefficients which strongly diverge as the dimensions of corresponding Hilbert subspaces $\mathcal{H}_{\mathcal{M}}$ become large. This is because the process transforming an element of $\mathcal{H}_{\mathcal{M}}$ to that of $\mathcal{H}_{\mathcal{M}'}$ with $\dim \mathcal{H}_{\mathcal{M}'} < \dim \mathcal{H}_{\mathcal{M}}$ becomes highly suppressed as $\dim \mathcal{H}_{\mathcal{M}}$ gets large (because of Eq. (7)). Let us now imagine regulating the sums of i as $\sum_i \rightarrow \sum_{i=1}^n$, in which case we can normalize all the solutions so that $\langle \Psi_I | \Psi_J \rangle = \delta_{IJ}$ for $I, J = 1, \dots, K$. We can then consider state $\rho = \sum_{I,J=1}^K d_{IJ} |\Psi_I\rangle \langle \Psi_J|$ with arbitrary finite positive semi-definite Hermitian matrix d_{IJ} , and calculate probabilities arising from ρ using a projection operator that selects (a finite number of) configurations compatible with some condition X : $\mathcal{O}_X = \sum_{i \in X} |\alpha_i\rangle \langle \alpha_i|$. The resulting probabilities are the same as those arising from $\rho' = \sum_{I,J=1}^N d_{IJ} |\Psi_I\rangle \langle \Psi_J|$, i.e. the state obtained by eliminating all the non-normalizable solutions from ρ , up to terms disappearing for $n \rightarrow \infty$. Therefore, the non-normalizable solutions can all be dropped from physical considerations.

The Hilbert space relevant for the multiverse $\mathcal{H}_{\text{Multiverse}}$, then, is spanned by the normalizable solutions to Eq. (23), and so is much smaller than \mathcal{H}_{QG} :

$$\mathcal{H}_{\text{Multiverse}} \subset \mathcal{H}_{\text{QG}}, \quad \dim \mathcal{H}_{\text{Multiverse}} \ll \dim \mathcal{H}_{\text{QG}}. \quad (30)$$

We note that this situation is analogous to usual quantum mechanical systems, e.g. a hydrogen atom. In the hydrogen atom, the state factor corresponding to a radial wavefunction $c(r)$ can be written as $|\psi\rangle = \int_0^\infty dr c(r) |r\rangle$. The only states relevant to physics of the hydrogen atom are

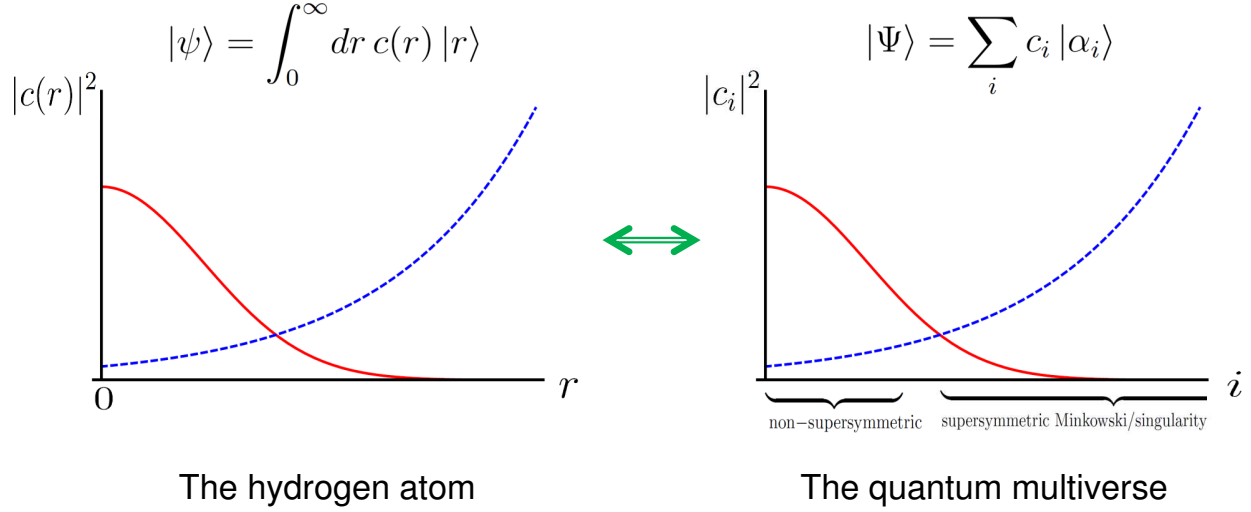


Figure 2: A schematic depiction of the analogy between the hydrogen atom and the quantum multiverse. In the case of the hydrogen atom, the only relevant states are those that satisfy the Schrödinger equation and are normalizable in the Hilbert space spanned by $|r\rangle$ (solid line); the non-normalizable modes are irrelevant (dashed line). In the quantum multiverse, the relevant states are those that satisfy Eq. (23) and are normalizable in Hilbert space \mathcal{H}_{QG} (solid line); the non-normalizable modes, which have diverging coefficients for supersymmetric Minkowski or singularity states, are irrelevant (dashed line).

those satisfying the normalizability condition $\int_0^\infty dr |c(r)|^2 < \infty$ in the Hilbert space spanned by $|r\rangle$. The other, non-normalizable solutions (which behave as $\ln c(r) \sim r$ at large r) are irrelevant. The situation in the quantum multiverse is similar. The non-normalizable solutions have infinitely strong supports in supersymmetric Minkowski vacua or singularity worlds, which have infinite-dimensional Hilbert spaces. These solutions, therefore, are irrelevant in making predictions in a “realistic world,” i.e. in a universe that has nonzero free energy. The only relevant states are those that are normalizable in the Hilbert space of quantum gravity, \mathcal{H}_{QG} . For a schematic drawing of this analogy, see Fig. 2.

5.3 The static multiverse states in \mathcal{H}_{QG}

We now discuss how our conditions Eqs. (22, 29) can be compatible with Eq. (8), which says that a generic multiverse state in \mathcal{H}_{QG} will evolve into a superposition of supersymmetric Minkowski and singularity states as $t \rightarrow \infty$. In order for Eq. (22) to be satisfied, the coefficients c_i of all the terms in $|\Psi(t)\rangle = |\Psi\rangle$ must be constant when expanded in components $|\alpha_i\rangle$. In a basis in which $|\alpha_i\rangle$ in \mathcal{H} have well-defined semi-classical configurations, the evolution operator $\exp(-iHt)$ (and thus H as well) is not diagonal. Therefore, the processes in Eq. (8) will occur for *generic* $|\Psi(t)\rangle$, but they must exactly be canceled by some “inverse processes” in $|\Psi\rangle$. In particular, in order for the

normalization condition in Eq. (29) to be satisfied, this must occur before the state is dissipated into infinite-dimensional Hilbert space.

Let us consider a physical configuration in a Minkowski universe in which there is a bubble wall surrounding us, which, however, is contracting toward us rather than expanding away. Such a configuration, which is exactly the time reversal of a usual expanding bubble configuration, is physically allowed, as the fundamental equation of the theory is symmetric under $t \rightarrow -t$. Usually, we do not consider this kind of configuration as it is only an exponentially small subset of all the configurations allowed by the theory; in particular, there is only an exponentially small probability for forming such a configuration starting from a generic, e.g. thermal, state. We are, however, now considering very special states, i.e. the states that satisfy Eqs. (22, 29), and in these states such configurations could balance the “loss” of semi-classically unstable states in Eq. (8). For example, the entire multiverse state is so “fine-tuned” that a reheating that occurs in a Minkowski universe produces exactly the configuration that puts the system back to (a superposition of) states in unstable vacua. Similar processes must also occur for singularities. Note that since these processes are exponentially suppressed under normal circumstances, they are invisible in the usual semi-classical analysis.

The states given by Eq. (29) are the ones in which all these and other processes are balanced.⁴ Since the inverse processes are unlikely to occur at the zero density, these states will explore only a finite-dimensional portion of Minkowski vacua (see the discussion at the end of Section 2.1). The number of independent states, therefore, may well be finite: $\dim \mathcal{H}_{\text{Multiverse}} < \infty$, which we will assume to be the case. Note that the sizes of various elements in H represented as a matrix acting on \mathcal{H}_{QG} differ significantly; in fact, they are expected to differ exponentially, or even double-exponentially, as some of the processes are highly suppressed. This implies that the values of $|c_i|$ ’s in Eq. (29) will also vary significantly. The resulting states $|\Psi\rangle$, therefore, are not excluded by corollary (*) in Section 3.1. The structure of $|\Psi\rangle$, and its consistency with observation, will be discussed further in Section 6.

5.4 Predictions in the static quantum multiverse

The number of independent normalizable solutions to Eq. (23) will depend on the structure of the multiverse, i.e. issue (a) in Section 2.3. In particular, the existence of a solution requires H to take a certain special form (so that it has at least one normalizable, zero-eigenvalue eigenvector), which we assume to be the case. Suppose there are N such solutions $|\Psi_I\rangle$ ($I = 1, \dots, N = \dim \mathcal{H}_{\text{Multiverse}} < \infty$). How can the physical predictions be made?

If $N = 1$, the multiverse state is simply $|\Psi\rangle \equiv |\Psi_1\rangle$. The probabilities are then given by the

⁴There is also the possibility that some (or all) of the states given by Eq. (29) do not contain any Minkowski or singularity components. This does not affect any of our discussions below.

generalized Born rule, Eq. (9), but now without the t integrals. (They simply give a constant factor $\int_{-\infty}^{+\infty} dt$, which cancels between the numerator and denominator.) The final formula is given by Eq. (2), which we reproduce here:

$$P(B|A) = \frac{\langle \Psi | \mathcal{O}_{A \cap B} | \Psi \rangle}{\langle \Psi | \mathcal{O}_A | \Psi \rangle}.$$

As discussed in Section 3.2, this formula can be used to answer any physical questions, including those about a system that we view as dynamically evolving.

In the case that $N > 1$, any multiverse states of the form $|\Psi\rangle = \sum_{I=1}^N c_I |\Psi_I\rangle$ or $\rho = \sum_{I,J=1}^N d_{IJ} |\Psi_I\rangle \langle \Psi_J|$ are allowed. In the absence of more information (or selection conditions), it is natural to assume that the multiverse is in the maximally mixed state

$$\rho = \frac{1}{N} \sum_{I=1}^N |\Psi_I\rangle \langle \Psi_I|, \quad (31)$$

where we have taken $|\Psi_I\rangle$'s to be orthonormal. This state is invariant under the basis change $|\Psi_I\rangle \rightarrow U_{IJ} |\Psi_J\rangle$, and is reduced to $|\Psi\rangle = |\Psi_1\rangle$ for $N = 1$. The probabilities are given by the mixed-state version of Eq. (2):

$$P(B|A) = \frac{\text{Tr}[\rho \mathcal{O}_{A \cap B}]}{\text{Tr}[\rho \mathcal{O}_A]}. \quad (32)$$

Note that Eq. (31), i.e. the maximally mixed state in $\mathcal{H}_{\text{Multiverse}}$, is different from Eq. (16), i.e. the maximally mixed state in \mathcal{H}_{QG} , in which the sum runs over all the possible states in \mathcal{H}_{QG} including the ones that do not satisfy Eq. (23). The state in Eq. (31), therefore, is not excluded by corollary (*) in Section 3.1.

6 Consistency with Observation

In this section we discuss the consistency of the present scenario with observation, specifically the observed arrow of time. Our approach here will be to allow for making assumptions on the structures of H and \mathcal{H}_{QG} (unless they are inconsistent with what we already know about string theory), and to see if the scenario is consistent. We do not claim that all of these assumptions are absolutely necessary—our purpose here is to argue that, despite its naive appearance, the scenario is not excluded by observation. More detailed analysis/modeling of the landscape will be left for future work.

6.1 The structure of \mathcal{H}_{QG}

Solutions to Eq. (23) depend on the structure of \mathcal{H}_{QG} as well as the form of H (and other operators). Here we assume that \mathcal{H}_{QG} contains only “cosmologically relevant” states. The minimally required

set of $\mathcal{H}_{\mathcal{M}}$'s that must be included in \mathcal{H} , i.e. in the right-hand side of Eq. (5), will then be those of FRW universes corresponding to all the possible vacua in the theory (and their straightforward generalizations, e.g. those of FRW universes with black holes). Not all spacetime must be contained in \mathcal{H} ; for example, \mathcal{H} need not contain a stable anti-de Sitter space without a singularity, which might only be a mathematical idealization because it does not arise through dynamical evolution in the FRW universes.

For each vacuum I of the theory, the number of states associated with an FRW universe in I is estimated as

$$\mathcal{N}_I = \sum_{n=\exp(\mathcal{A}_{I,\min}/2)}^{\exp(\mathcal{A}_{I,\max}/2)} n \simeq \frac{1}{2} e^{\mathcal{A}_{I,\max}}, \quad (33)$$

where $\mathcal{A}_{I,\min}$ and $\mathcal{A}_{I,\max}$ are the minimum and maximum areas of the apparent horizon in this universe, and we have used $\mathcal{A}_{I,\max} \gg \mathcal{A}_{I,\min}$ in the last equation. While possible deformations of the apparent horizon, e.g. by the existence of black holes, can have corrections to the explicit expression, we expect that the above estimate gives a qualitatively correct result: $\ln \mathcal{N}_I \approx O(\mathcal{A}_{I,\max})$. The area $\mathcal{A}_{I,\max}$ is given by the inverse of the absolute value of the vacuum energy density (in Planck units) $\mathcal{A}_{I,\max} \sim 1/|\rho_{\Lambda,I}|$, since in a de Sitter universe the apparent horizon approaches the event horizon at late times, while in an anti-de Sitter universe it has the maximum area when p hits the singularity at $t \sim 1/|\rho_{\Lambda,I}|^{1/2}$. We therefore find

$$\ln \mathcal{N}_I \sim \frac{1}{|\rho_{\Lambda,I}|}. \quad (34)$$

This implies that the number of states associated with a vacuum with $\rho_{\Lambda,I} \neq 0$ is finite.

6.2 The arrow of time in the static multiverse

We now consider a solution to the equation $H|\Psi\rangle = 0$, a part of the conditions in Eq. (23). We can view this equation as requiring that $|\Psi\rangle$ is in a stationary state in \mathcal{H}_{QG} . (In fact, the equation is stronger than that, since the eigenvalue of H must be zero.) In particular, it implies that the probability current creating states in vacuum I must be balanced with that destroying those for each I (in fact, each state in I). At the semi-classical level, this condition is impossible to satisfy for terminal vacua. As discussed in Section 5.3, however, our state is special, obtained after solving the “quantization condition” $H|\Psi\rangle = 0$, so that it can also be satisfied for these vacua.

Let us now consider vacuum J that can support any observer, either an ordinary observer or a Boltzmann brain. We will argue that the arrow of time is predicted if the following three conditions are met for all possible J 's:

- (I) Transitions to states in J from those in other vacua are mainly through the states having low coarse-grained entropies in J , i.e. elements of $\mathcal{H}_{\mathcal{M}}$ with $\ln \dim \mathcal{H}_{\mathcal{M}} \ll \mathcal{A}_{J,\max}$.

- (II) Subsequent evolution in vacuum J produces ordinary observers with probability ϵ_J , which may be suppressed exponentially but not double-exponentially.
- (III) The rate of producing Boltzmann brains $\Gamma_{\text{BB},J}$ in vacuum J , which is double-exponentially suppressed (see, e.g. [19]), is smaller than the decay rate Γ_J of the vacuum itself.

Namely, if the structure of H is such that it satisfies all these conditions, then the scenario is compatible with observation. (The “transitions” and “evolution” here, of course, refer to the apparent ones in $|\Psi\rangle$, which is in itself static.)

To see this, let us consider the distribution of the size of the coefficients $|c_i^J|$ of various terms in $|\Psi\rangle$ corresponding to the states in vacuum J , $|\alpha_i^J\rangle$. For this purpose, we define the quantity P_τ^J corresponding to the probability for a universe to be at FRW time t_{FRW} between τ and $\tau + d\tau$:

$$P_\tau^J d\tau = \sum_{i|\tau < t_{\text{FRW}} < \tau + d\tau} |c_i^J|^2, \quad (35)$$

where t_{FRW} should be specified by physical configurations in $|\alpha_i^J\rangle$. The distribution of P_τ^J then follows from the definition of Γ_J :

$$P_\tau^J = P_0^J e^{-\Gamma_J \tau}, \quad (36)$$

where we have assumed that the transitions to states in J occur at $\tau = 0$ either through Coleman-De Luccia [20] or Hawking-Moss [21] processes (or their inverses), although our conclusion is insensitive to this assumption. Note that in these cases it is indeed natural to expect that states just after the transitions are the ones having low coarse-grained entropies, i.e. in $\mathcal{H}_{\mathcal{M}}$ with $\ln \dim \mathcal{H}_{\mathcal{M}} \ll \mathcal{A}_{J,\text{max}}$, because both the start and end points of the Coleman-De Luccia tunneling in field space are away from local minima (if the false vacuum has a positive vacuum energy), and the Hawking-Moss transition is a thermal process occurring through the field climbing up the potential barrier [22].

Now, the definitions of ϵ_J and $\Gamma_{\text{BB},J}$ in (II) and (III) above imply that if we compute the probability of $|\Psi\rangle$ containing ordinary observers (OO) or Boltzmann brains (BB) in vacuum J using the corresponding projection operators $\mathcal{O}_{\text{OO},J}$ and $\mathcal{O}_{\text{BB},J}$, then we obtain

$$\langle \Psi | \mathcal{O}_{\text{OO},J} | \Psi \rangle \sim \epsilon_J P_0^J, \quad (37)$$

$$\langle \Psi | \mathcal{O}_{\text{BB},J} | \Psi \rangle \sim \Gamma_{\text{BB},J} \int P_\tau^J d\tau = \frac{\Gamma_{\text{BB},J}}{\Gamma_J} P_0^J. \quad (38)$$

Here, the projection operators select observers in a specific range of location and velocity with respect to p , although the results do not depend on the chosen location or velocity because of Eq. (23). Under conditions (II) and (III), this gives

$$\frac{\langle \Psi | \mathcal{O}_{\text{BB},J} | \Psi \rangle}{\langle \Psi | \mathcal{O}_{\text{OO},J} | \Psi \rangle} \sim \frac{\Gamma_{\text{BB},J}}{\epsilon_J \Gamma_J} \lll 1, \quad (39)$$

where we have used the fact that $\Gamma_{\text{BB},J}$ is double-exponentially suppressed while ϵ_J is not. (In fact, we only need $\epsilon_J > \Gamma_{\text{BB},J}/\Gamma_J$ to obtain this result, so ϵ_J may be double-exponentially suppressed.) We therefore find that the overwhelming majority of observers are indeed ordinary observers, and thus perceive time’s arrow (as discussed in Section 4.2).

Perhaps not surprisingly, the conditions described above are similar to the ones obtained in Ref. [14] in the context of the evolving multiverse, despite the fact that the overall physical pictures are rather different. One distinct feature of the present scenario in this respect is that since there is no “initial vacuum,” the absolute nonexistence of Boltzmann brains in such a vacuum ($\Gamma_{\text{BB},*} = 0$ in the notation of Ref. [14]) need not be imposed. In any case, as discussed in Ref. [14], the conditions described above, in particular (I), are likely to be satisfied in the string landscape. It is, therefore, quite promising that the scenario discussed in this paper is indeed consistent with observation in the realistic string theory setup.

7 Discussions

In this paper we have studied the multiverse in the quantum mechanical framework recently proposed in Refs. [1, 2]. By requiring that the laws of quantum mechanics are not violated (Hypothesis I) and that physical predictions do not depend on the reference frame one chooses to describe the multiverse (Hypothesis II), we have found that the multiverse state must be static; in particular, the multiverse does not have a beginning or end.

Despite its naive appearance, the scenario does not contradict observation, including the fact that we observe that time flows in a definite direction. *The arrow of time is simply an emergent phenomenon that is occurring in the branch (terms) corresponding to our universe in the static multiverse state*—the terms that would be obtained by evolving the system from lower entropy states have much larger coefficients than the terms that cannot. The scenario is summarized by the selection conditions in Eq. (23), imposed on the states in \mathcal{H}_{QG} . With these conditions, any multiversal questions can be answered using the Born rule, Eq. (2) or (32), *without any additional input*, once the explicit form of the operators such as H is known. This scenario, therefore, provides a completion of the framework of the quantum multiverse in Refs. [1, 2].

The supplementary condition of the form $H|\Psi\rangle = 0$ has certainly been considered before—indeed, this is nothing but the well-known Wheeler-DeWitt equation [17]. The scenario presented here, however, differs from standard applications of this equation in several important ways:

- The redundancies associated with gravity are much larger than what are usually imagined. In particular, they reduce the Hilbert space in such a way that it contains only the spacetime region within the reference point’s (stretched) apparent horizon [1, 2]. This is important to avoid ambiguities associated with eternally inflating spacetime. The ultimate origin of these

large redundancies will, presumably, be string theory.

- We apply the supplementary conditions corresponding to the whole set of time translation and reference frame changes with zero global charges, even if the universe is not closed. Since spacetime is defined only within the apparent horizon, this requires the assumption on the structure of the theory on this surface, which is intrinsically quantum mechanical. Note that it is this assumption that is responsible for the static nature of the multiverse state, which in turn excludes the possibility for the multiverse to have a beginning or end.
- We analyze the consequences of the supplementary conditions at the microscopic level. This selects very special states that are not visible in the analysis at the semi-classical level. In fact, normalizable solutions to the conditions correspond to the states in which the processes of Eq. (8) are balanced with the inverse processes, which put the system back from terminal vacua to unstable vacua.

It is quite satisfying that such simple requirements as Hypotheses I and II lead to a consistent and predictive scheme for the entire multiverse.

Finally, it is instructive to draw a close analogy between the situation in the quantum multiverse described here and that in the standard, hydrogen atom. As is well known, the hydrogen atom cannot be correctly described using classical mechanics. Any orbit of the electron is unstable with respect to the emission of synchrotron radiation. Even if we artificially ignore the emission, the electron can orbit the nucleus at an arbitrary radius, unable to explain the discrete spectral lines. The solution to these problems is intrinsically quantum mechanical, i.e. quantum mechanics is responsible for the very existence of the hydrogen atom, not just providing a correction to the classical picture.

The situation in the quantum multiverse is similar. At the semi-classical level, the multiverse is unstable to the decay to terminal states, such as supersymmetric Minkowski vacua and singularities. Even if we artificially ignore the process of vacuum decays, it would lead to phenomena such as Poincaré recurrence, contradicting observation (the dominance of Boltzmann brains). The picture presented here says that the solution to these problems is *intrinsically quantum mechanical*—one cannot see it in the usual semi-classical analysis. The multiverse state is very special: a normalizable state satisfying the “quantization conditions” of Eq. (23), as in the case of the hydrogen atom. In the case of the hydrogen atom, these conditions make the dimension of Hilbert space from continuous infinity $\psi(r, \theta, \varphi)$ to countable infinity (n, l, m) . In the quantum multiverse, they will presumably make it from countable infinity to finite: $\dim \mathcal{H}_{\text{QG}} \rightarrow \dim \mathcal{H}_{\text{Multiverse}}$.

After all, quantum mechanics treats the multiverse very similarly to the hydrogen atom. Our job is then to figure out the precise structure of the multiverse, a system which we are a part of. Hopefully, further progress in string theory will serve this purpose.

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References

- [1] Y. Nomura, JHEP **11**, 063 (2011) [arXiv:1104.2324 [hep-th]].
- [2] Y. Nomura, arXiv:1110.4630 [hep-th].
- [3] A. H. Guth and E. J. Weinberg, Nucl. Phys. B **212**, 321 (1983); A. Vilenkin, Phys. Rev. D **27**, 2848 (1983); A. D. Linde, Phys. Lett. B **175**, 395 (1986); Mod. Phys. Lett. A **1**, 81 (1986).
- [4] For reviews, see e.g. A. H. Guth, Phys. Rept. **333**, 555 (2000) [arXiv:astro-ph/0002156]; A. Vilenkin, J. Phys. A **40**, 6777 (2007) [arXiv:hep-th/0609193]; S. Winitzki, Lect. Notes Phys. **738**, 157 (2008) [arXiv:gr-qc/0612164]; A. Linde, Lect. Notes Phys. **738**, 1 (2008) [arXiv:0705.0164 [hep-th]].
- [5] Y. Nomura, AstRv. **7**, 36 (2012) [arXiv:1205.2675 [hep-th]].
- [6] A. Mithani and A. Vilenkin, arXiv:1204.4658 [hep-th]; L. Susskind, arXiv:1204.5385 [hep-th]; arXiv:1205.0589 [hep-th].
- [7] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D **48**, 3743 (1993) [arXiv:hep-th/9306069]; L. Susskind, Phys. Rev. Lett. **71**, 2367 (1993) [arXiv:hep-th/9307168]; C. R. Stephens, G. 't Hooft and B. F. Whiting, Class. Quant. Grav. **11**, 621 (1994) [arXiv:gr-qc/9310006].
- [8] G. 't Hooft, arXiv:gr-qc/9310026; L. Susskind, J. Math. Phys. **36**, 6377 (1995) [arXiv:hep-th/9409089]; R. Bousso, JHEP **07**, 004 (1999) [arXiv:hep-th/9905177].
- [9] R. Bousso and J. Polchinski, JHEP **06**, 006 (2000) [arXiv:hep-th/0004134]; S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240]; L. Susskind, arXiv:hep-th/0302219; M. R. Douglas, JHEP **05**, 046 (2003) [arXiv:hep-th/0303194].
- [10] See, e.g., R. Bousso, Rev. Mod. Phys. **74**, 825 (2002) [hep-th/0203101], Section VII A.
- [11] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
- [12] G. Larsen, Y. Nomura and H. L. L. Roberts, Phys. Rev. D **84**, 123512 (2011) [arXiv:1107.3556 [hep-th]].

- [13] L. Dyson, M. Kleban and L. Susskind, JHEP **10**, 011 (2002) [arXiv:hep-th/0208013].
- [14] R. Bousso, arXiv:1112.3341 [hep-th].
- [15] H. Ollivier, D. Poulin and W. H. Zurek, Phys. Rev. Lett. **93**, 220401 (2004) [arXiv:quant-ph/0307229]; R. Blume-Kohout and W. H. Zurek, Phys. Rev. A **73**, 062310 (2006) [arXiv:quant-ph/0505031].
- [16] P. A. M. Dirac, *Lectures on Quantum Mechanics*, (Belfer Graduate School of Science, Yeshiva University, New York, 1964).
- [17] B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).
- [18] R. Arnowitt, S. Deser and C. W. Misner, in *Gravitation: An Introduction to Current Research*, ed. L. Witten (John Wiley & Sons, New York, 1962) [gr-qc/0405109].
- [19] R. Bousso, B. Freivogel and I-S. Yang, Phys. Rev. D **79**, 063513 (2009) [arXiv:0808.3770 [hep-th]]; A. De Simone, A. H. Guth, A. Linde, M. Noorbala, M. P. Salem and A. Vilenkin, Phys. Rev. D **82**, 063520 (2010) [arXiv:0808.3778 [hep-th]].
- [20] S. R. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
- [21] S. W. Hawking and I. G. Moss, Phys. Lett. B **110**, 35 (1982).
- [22] E. J. Weinberg, Phys. Rev. Lett. **98**, 251303 (2007) [hep-th/0612146].